Key

2.5 Completing the Square

Objectives:

Use completing the square to solve a quadratic equation

PST: Two same Factors $(x-3)(x-3) \rightarrow (x-3)^2$

Ex 1. Perfect square trinomials

Complete the square for each quadratic expression to form a perfect-square trinomial and factor. $\left(\frac{1}{2}b\right)^2$ half b squared

a) $x^2 - 10x$ $\left(-10 \cdot \frac{1}{2}\right)^2$ $(-5)^2$ 25 $x^2-10x+25$ $(x-5)^2$

b) $x^2 + 27x$ $\left(\frac{1}{2} \cdot 27\right)^2 = \left(\frac{27}{2}\right)^2 = (13.5)^2$ 182.25 $x^2+27x+182.25$ $(x+13.5)^2$

Practice Do NOW

Complete the square for each quadratic expression to form a perfect-square trinomial. Then write the new expression as a binomial squared.

1) $x^2 - 7x$ $\left(\frac{-7}{2}\right)^2$ $(-3.5)^2$ $x^2-7x+12.25$ $(x-3.5)^2$

2) $x^2 + 16x$ $\left(\frac{16}{2}\right)^2 = 8^2 = 64$ $x^2+16x+64$ $(x+8)^2$

Solving Quadratic Equations by Completing the Square

$$x^2 + 8x - 20 = 0$$

Step 1: Move Bad C Move quadratic term, and linear term to left side of the equation and c to the other side

$x^2+8x = 20$

Step 2: Find new c. Create a PST by $(b/2)^2$ and adding it to both sides of the equation. $\left(\frac{1}{2} \cdot 8\right)^2$ 4^2 16 $x^2+8x+16 = 20+16$

Step 3: Factor the PST and simplify the other side

$(x+4)^2 = 36$

Step 4: Take the square root of each side

$x+4 = \pm 6$

Step 5: Simplify the square root. Set up the two possibilities and solve.

$x+4=6$ $x+4=-6$

$x=2$ $x=-10$

Example 2

Solve $x^2 + 8x + 5 = 0$ by completing the square.

$\left(\frac{8}{2}\right)^2$

4^2

16

$x^2+8x = -5$

$x^2+8x+16 = -5+16$

$(x+4)^2 = 11$

$x+4 = \pm\sqrt{11}$

$x = \sqrt{11} - 4$ or $x = -\sqrt{11} - 4$

Example 3. Solve $x^2 - 2x + 3 = 0$ by completing the square.

$\left(\frac{-2}{2}\right)^2$

1

$x^2-2x = -3$

$x^2-2x+1 = -3+1$

$(x-1)^2 = -2$

$x-1 = \pm\sqrt{2}\,i$

$x = 1 \pm \sqrt{2}\,i$

Extra Example 3

Solve $3x^2 - 6x = 5$ by completing the square with an "a".

$3(x^2-2x+\_\_\_) = 5$

$\left(\frac{-2}{2}\right)^2$

1^2

1

$3(x^2-2x+1) = 5+3$ multiply (1)(3) to balance

$3(x-1)^2 = 8$

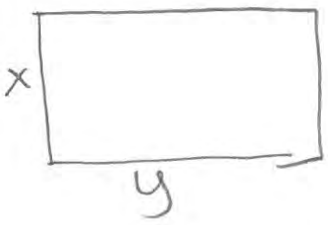
$(x-1)^2 = 8/3$

$x-1 = \frac{\pm\sqrt{8}}{\pm\sqrt{3}}$

$x = \frac{\pm 2\sqrt{2}}{\sqrt{3}} + 1$

$x = \pm\frac{2\sqrt{6}}{3} + 1$

Example 4. Libby plans to create a rectangular pasturing enclosure. She has 340 m of fencing available for the enclosure's perimeter and wants it to have an area of 6,000 m^2. What dimensions should Libby use?



$P = 340$

$2x+2y = 340$

$x+y = 170$

$x = 170-y$

$A = 6000$

$xy = 6000$

$(170-y)(y) = 6000$

$170y - y^2 = 6000$

$y^2 - 170y = -6000$

$\left(\frac{-170}{2}\right)^2$

85^2

$y^2-170y+7225 = -6000+7225$

$(y-85)^2 = 1225$

$y-85 = \pm 35$

$y = 120$ or 50

$x = 50$ $x = 120$

Example 5. Write a quadratic in vertex form.

$y = -2x^2 + 10x + 1$

$\left(\frac{-5}{2}\right)^2 = \frac{25}{4} = 6.25$

$y - 1 = -2(x^2 - 5x + 6.25)$

$y - 1 + (6.25)(-2) = -2(x - 2.5)^2$

$y - 13.5 = -2(x - 2.5)^2$

$y = -2(x - 2.5)^2 + 13.5$

You try.

$y = -3x^2 - 9x + 7$

$y = 2x^2 + 12x + 9$

$\left(\frac{6}{2}\right)^2 = 9$

$y - 9 = 2x^2 + 12x$

$y - 9 = 2(x^2 + 6x + 9)$

$y + 9 = 2(x+3)^2$

$y = 2(x+3)^2 - 9$